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On Exact Tachyon Potential in Open String Field Theory

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In these notes we revisit the tachyon lagrangian in the open string field theory using background independent approach of Witten from 1992. We claim that the tree level lagrangian (up to second order in derivatives and modulo some class of field redefinitions) is given by $L = e^{-T}(\partial T)^2 + (1+T)e^{-T}$. Upon obvious change of variables this leads to the potential energy $-\phi^2 \log \frac{\phi^2}{e}$ with canonical kinetic term. This lagrangian may be also obtained from the effective tachyon lagrangian of the p-adic strings in the limit $p \to 1$. Applications to the problem of tachyon condensation are discussed.

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1. Introduction

The problem of tachyon condensation attracted wide interest in the recent string theory literature after the proposal of A. Sen [1]. In [2] it was argued that the tachyon potential takes the form:

$$V(T) = Mf(T), \tag{1.1}$$

with M-mass of D-brane and f - universal function independent of the background where brane is embedded. The conjecture of Sen states that $f(T)$ has a stationary point (local minimum) at some $T = T_c < \infty$ such that

$$f(T_c) = -1. \tag{1.2}$$

Thus total mass vanishes: $M + V(T_c) = M(1 + V(T_c)) = 0$.

Several arguments in favor of this conjecture were proposed.

It has been demonstrated in [1], [3] that sting field theory action of Witten from 1986 [4] and the systematic approximation scheme of Kostelecky and Samuel [5] may be successfully used for verifying the Sen's conjecture. For example in *level zero* approximation:

$$f^0(t) = 2\pi^2(-\frac{1}{2}t^2 + \frac{1}{3}\frac{t^3}{r}), \qquad r = \frac{4}{3\sqrt{3}}. \tag{1.3}$$

This function has local minimum at $t = t_c = r^3 = 0.456$ with the value $f(t_c) = -0.684$, which confirms the conjecture with 70% of accuracy. This approximation has been further improved and indeed it seems that the value of f approaches -1 (though there is no obvious small parameter expansion in level truncation method).

In [6] the case of the p-adic strings was discussed. It was shown that the old results on the effective field theory of the tachyon in the p-adic string theory [7] could provide an interesting example for the discussion of the tachyon condensation. In [8] the effective field theory of the tachyon which reproduce the tachyon scattering amplitudes in p-adic string theory was constructed. The lagrangian from [8] has the form:

$$S(\phi) = \frac{1}{g^2}\frac{p^2}{p-1}\int d\sigma(-\frac{1}{2}\phi p^{-\frac{1}{2}\triangle}\phi + \frac{1}{p+1}\phi^{p+1}), \tag{1.4}$$

with the equations of motion:

$$p^{-\frac{1}{2}\triangle}\phi = \phi^p. \tag{1.5}$$

In this paper we would like to approach the problem from the point of view of two-dimensional theories living on the world-sheet of the strings (for related recent discussions see [9]) . For simplicity we are dealing with the open strings in flat 26th dimensional space.

To verify the Sen scenario of tachyon condensation one needs to have the definition of the string theory off-shell. It is well known that the off-shell continuation of the theory suffers from the field redefinition ambiguity (see for example [10]). We partially fix this ambiguity by looking at the concrete recipe of the off-shell definition of the string theory action proposed by Witten in [11] and obtain the following effective lagrangian up to the second derivatives in the tachyon field:

$$L = e^{-T}(\partial T)^2 + (1+T)e^{-T}. \tag{1.6}$$

It is natural to make a field redefinition to have a derivative term of standard form. This gives the following form of the tachyon potential:

$$V(\phi) = -\phi^2 \log \frac{\phi^2}{e}. \tag{1.7}$$

Thus we appear to obtain a lagrangian which shows up in many interesting problems of theoretical physics for several decades and string theory in particular.

Surprisingly it could be obtained from the effective lagrangian of the tachyon in the p-adic string theory in the formal limit $p \to 1$. Expanding the equation (1.5) around $p = 1$ and looking at the linear term we get:

$$\triangle\phi = 2\phi \log \phi. \tag{1.8}$$

Taking this lagrangian seriously leads to an interesting conclusion of the fate of the tachyon in open string theory. The potential energy has the stable vacuum on the boundary of the configuration space at $T = \infty$ or $\phi = 0$. Around this new vacuum the mass of the tachyon excitations is infinite. This may be a manifestation of the disappearance of the whole tower of open string fields at this point and confirms some of the ideas from [1].

Obviously taking another off-shell continuation could change the tachyon lagrangian substantially. We hope however that the qualitative picture of the "new" tachyon vacuum will provide an interesting scenario for the tachyon condensation.

Unexpected connection with the p-adic string also supports our result (1.6).

2. Background Independent Open String Field Theory

Let us first remind the definition given by Witten for background independent open string field theory [11]. This action is a functional of boundary perturbations (corresponding couplings) of bulk CFT:

$$dS =< d \int_{\partial D} \mathcal{O}\{Q, \int_{\partial D} \mathcal{O}\} >, \tag{2.1}$$

where one considers the disk D with boundary ∂D, conformal field theory on D (for simplicity we will take this CFT to be just bosonic 26 dimensional string) perturbed with arbitrary closed string operator $b_{-1} \int_C O$ integrated over the contour C with contour approaching the boundary of the disk ∂D; $Q = \int_C j_{BRST}$, and again contour C approaches the boundary. This also can be written in the BV formalism as

$$dS = i_V \omega, \tag{2.2}$$

where ω is odd-symplectic structure of BV formalism and V is a vector field that generates the symmetries of ω. This action has obvious property that it has critical points exactly when boundary pertrubation is exactly marginal. The general formula for this action has been found in [12] where it was demonstrated that:

$$S = -\beta^i \partial_i Z + Z, \tag{2.3}$$

with Z -partition function and β^i - beta function for coupling t^i. For the simplest, quadratic, boundary perturbation

$$\mathcal{O} = cT, \qquad T = \frac{T_0}{2\pi} + \sum_i \frac{u_i}{8\pi} X_i^2, \tag{2.4}$$

the action was computed in the second paper of [11]:

$$S = (-\sum_j u_j \frac{\partial}{\partial u_j} - (T_0 + \sum_j u_j) \frac{\partial}{\partial T_0} + 1) Z, \tag{2.5}$$

with

$$Z = e^{-T_0} \prod_i \sqrt{u}_i e^{\gamma u_i} \Gamma(u_i). \tag{2.6}$$

Simple calculations show that the asymptotic of the partition function has the form:

$$Z = \sqrt{\frac{\pi}{4}} \int dX e^{-T_0 - \frac{u}{4} X^2} (1 + O(u^2)). \tag{2.7}$$

In order to study the tachyon potential it is enough to set $u_i = 0$ and we find (up to the divergent factor - in the limit $u \to 0$ there is no partition function rather one has partition function per unit volume; for discussion regarding this see [11]):

$$V(T_0) = (1 + T_0)e^{-T_0}. \tag{2.8}$$

This function gives exact tree level tachyon potential when all other fields are set to zero (corrections involve derivatives of tachyon and all other fields which we have ignored at the moment) in a specific coordinate system and regularisation dictated by world-sheet boundary sigma model approach. It is obviously different than the potential $Mf^0(t)$ from (1.3), and although we can shift the potential by $const = -1$ in order to have zero at $T = 0$ and at the same time get the value at the minimum $V(T_c) = -1$ this minimum is not saturated and is at infinity $T = \infty$ which is very different compared to Sen's conjecture.

Before we turn to the derivation of terms involving derivatives of arbitrary tachyon field $T(X)$ in string field theory lagrangian we would like to make some general remarks about the consequences of the field redefinition for the tachyon lagrangian. We will discuss only the effect of the field redefinition which contain at most the two derivatives. Let us start with simple (non derivative) form of the tachyon field redefinition:

$$T \to \widetilde{T} = f(T), \tag{2.9}$$

in the action functional:

$$S(T) = \int (h(T)(\partial T)^2 + V(T) + \cdots). \tag{2.10}$$

We have:

$$\widetilde{S}(T) = \int (h(f(T))(\frac{\partial f(T)}{\partial T})^2(\partial T)^2 + V(f(T)) + \cdots). \tag{2.11}$$

The equation for the critical points of the potential is:

$$\frac{\partial f(T)}{\partial T}\frac{\partial V}{\partial T}(f(T)) = 0 \tag{2.12}$$

It is clear that the possible "new" fixed points come from the zeros of the Jacobian of the field transformation (singular change of variables). Therefore from (2.10) we conclude that in the "new" fixed points the coefficient (metric) in front of kinetic term is zero (if it was non-singular before the field redefinition).

However there is no reason to believe that we should not consider more general tachyon field redefinition:[1]

$$T \to \widetilde{T} = f(T) + g(T)(\partial T)^2 + \cdots, \tag{2.13}$$

Under this transformation the new coefficient functions in the lagrangian are:

$$\widetilde{V}(T) = V(f(T)), \tag{2.14}$$

$$\widetilde{h}(T) = h(f(T))(\partial f(T))^2 + V'(f(T))g(T), \tag{2.15}$$

and we see that even if there was no kinetic term in original action it has been created: $L_{kin} = V'(f(T))g(T)(\partial T)^2$. For our potential in (2.8) this gives $L_{kin} = -g(T)Te^{-T}(\partial T)^2$ for $f(T) = T$.

In general nothing specific could be said about the new parameters of the lagrangian. However adding some additional information from boundary sigma model approach we could extract useful information, because after fixing the regularisation scheme (treatment of contact terms) in the definition via sigma model the action in (2.1) is uniquely defined (up to a constant).

The kinetic term in the action defines the measure (metric in field space) in second quantized path integral, so, for instance fixing the boundary field theory analog of the Zamolodchikov metric to have the simple exponential form $G_{TT} \sim e^{-T}$ (up to same divergent factor which enters in (2.8) and higher derivative corrections) we come to the conclusion that kinetic term in our variables T is:

$$L_{kin} \sim e^{-T}(\partial T)^2. \tag{2.16}$$

Indeed this turnes out to be the right answer and, as we show in a moment, it follows directly from (2.1), (2.3); (it can be also confirmed in standard sigma model approach [14]).

Thus we would like to show that tachyon action up to two derivatives in T is:

$$S(T) = \int e^{-T}((\partial T)^2 + (T+1)). \tag{2.17}$$

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[1] Regarding recent discussions of important physical role of field redefinition and also full list of references on the subject of tachyon condensation see [13].

According (2.13) this action is a special case of the the whole family of the tachyonic actions

$$S_*(T) \sim \int e^{-T}((1+g(T)T)(\partial T)^2 + (T+1)) \tag{2.18}$$

generated by the field redefinitions

$$T \to T - g(T)(\partial T)^2 + ... \tag{2.19}$$

with ... denoting the higher derivatives of tachyon field. In addition one can also consider action of ordinary $Diff$ group. Obviously we study only the perturbative class of equivalence of this action (e.g. with non-perturbative $g(T) = -\frac{1}{T}$ one can totally remove kinetic term from our action (2.17)). It seems that we should think about the action (2.17) as the one particular form which may be extracted from the sigma-model.

Since we have the general expression for the action (2.3) we can apply it to the case of boundary perturbation with only tachyon field turned on: $\mathcal{O} = c(\theta)T(X(\theta))$. In this case we know following expressions for β-function and partition function:

$$\beta^T(X) = (1+2\Delta)T(X) + a_1(T)\partial T + a_2(T)\partial^2 T + a_3(T)(\partial T)^2 + ..., \tag{2.20}$$

$$Z = \int dX e^{-T(X)}(1 + b(T)(\partial T)^2 + ...), \tag{2.21}$$

with

$$a_1(0) = 0, \quad a_2(0) = 0, \quad a_3(0) = const, \quad b(0) = const, \tag{2.22}$$

and all these coefficients are given by some concrete power series expression in T which can be determined from boundary sigma model. In fact by comparing (2.6) with the explicit formula (2.7) one could say more. Taking into account the simple dependence of the action on the tachyon zero mode we conclude that b does not depend on the tachyon field. Then checking the (2.21) against (2.7) gives us $b = 0$.

Properties (2.22) just explain that (2.20), (2.21) are perturbative expansions in tachyon and its derivatives with leading terms determined by free field computations on world-sheet (note that for the case studied in [11] these properties are obviously satisfied). Thus we have:

$$S = -\int dX \beta^T(X)\partial_{T(X)} Z(T) + Z(T), \tag{2.23}$$

with $\beta^T(X)$ and $Z(T)$ expressed in terms of (2.20) and (2.21).

Interesting fact about the formula (2.3) is that it encodes equations of motion in two ways: 1. equations of motion are derived in stanard way from the action: $dS = 0$, 2. equations of motion derived in 1. are satisfied by zeros of vector field in (2.3): $\beta^i = 0$.

After some simple algebra we find from (2.3) and (2.20), (2.21), (2.23):

$$S = \int e^{-T}[(2 + Q(T))(\partial T)^2 + (T + 1)], \tag{2.24}$$

where $Q(T)$ is linear combination of a_i's and b and their derivatives with respect to T.

$$Q(T) = -b + T(b - b') + (a_2 - a_2') + a_3 \tag{2.25}$$

Now we would like to show that $Q(0) = -1$.

We use two ways of looking on equations of motion mentioned above. Beta function equation in linear approximation is just free tachyon equation $2\Delta T + T = 0$; the minimum of action (2.24), again in linear approximation, is given by $4\Delta T + 2Q(0)\Delta T + T = 0$. These are consistent only if $Q(0) = -1$, so we can write: $Q(T) = -1 + P(T)T$. We shall note that in quadratic boundary perturbation (2.5) we had $a_1 = a_2 = 0$ and $a_3 = -1$. This is consistent with the general expression (2.25).

We conclude that string field theory action (2.1), (2.3), (2.24) is in the equivalence class (2.19) of action (2.17), with $g(T) = P(T)$. Explicit form of $P(T)$ depends on regularisation scheme adopted in boundary sigma model. Simple analysis of the dependence on zero tachyon mode allows to fix the function Q uniquely - background independent open string theory naturally leads to the constant function $Q(T) = -1$.

Interestingly, for the form (2.17) on the equations of motion:

$$e^{-T}(2\Delta T - (\partial T)^2 + T) = 0 \tag{2.26}$$

the whole action can be simply written as $\int e^{-T}$ (we just integrate equation (2.26) over spacetime) and we learn that on-shell action and partition function do indeed coincide (in the approximation adopted throught this paper). Also we see that together with beta function equation from (2.20) and partition function (2.21) we get: $b = 0, a_1 = a_2 = 0, a_3 = -1$. This completes our derivation of (2.17).

This kinetic term for the tachyon has non standard form. In particular it has zero at infinity in T variables. Taking into account the obvious field redefinition (element of remaining $Diff$):

$$e^{-T} = \phi^2 \tag{2.27}$$

we obtain the final action announced in the Abstract:

$$S \sim \int 4(\partial\phi)^2 - \phi^2 \log \frac{\phi^2}{e}. \tag{2.28}$$

This field redefinition is singular at $T = \infty$ or same at $\phi = 0$. We will comment about the relation between the coordinates used in sigma model approach and Chern-Simons string field theory at the end of the paper.

We shall note that in the minimum of potential energy V of derived action (2.17) (which is at $T = \infty$) metric e^{-T} has zero in accordance with the conjecture of Sen. But difference is that in the variables T of our approach this minimum is at infinity and is never saturated (it is on the boundary of configuration space); it is not surprising that variable T differs from t used in (1.3), but certainly some relation can be established (see next section).

The appealing property of this potential is that the effective mass of the tachyon excitations around the new minimum at $T = \infty$ is infinite. One could hope that similar mechanism gives the infinite masses to other open string excitations but this deserves further investigations.

3. Some comments regarding the relation to level approximation scheme in string field theory.

Now we would like to make some preliminary remarks on connection of the field variables used in our sigma model analysis and the field variables used in **CS** string field theory for analysis of the tachyon potential in the level approximation scheme [5],[3]. Obviously these parameterizations of the string field functional are quite different. In particular the gauge (BRST) transformation acts differently on these variables which is most obvious for U(1) gauge fields [2]. In the sigma model approach the gauge transformation is independent on background fields while in **CS** string field theory it is linear in string fields. The perturbative solution for this field redefinition enters in essential way in the level-approximation scheme of [5].

Below we outline the construction of this field redefinition in terms of the 2d functional integral. To define this field redefinition we give the expression for the wave function in string field theory parameterized by the sigma model fields.

[2] For recent discussion see e.g. [15]

Consider 2d field theory on the disk with the bulk action S_{Bulk} describing the closed strings in the flat 26d background. Divide the boundary on two equal parts I_1 and I_2. On I_1 we take the boundary conditions $X(\sigma) = X_*(\sigma)$ with some fixed $X_*(\sigma)$ playing the role of string wave function argument. On the other part I_2 of the boundary we consider the free boundary conditions but with the boundary action parameterized by the sigma model variables:

$$S_{bound} = -\int_{I_2} d\sigma (T(\sigma) + A_\mu \partial X^\mu + \cdots) \tag{3.1}$$

The proposed parameterizations of the open string wave function is given by the following condition on variations:

$$\delta\Psi(X_*(\sigma)) \sim \int DX(\sigma) e^{S_{bound}+S_{bulk}} (\int_{I_2} d\sigma (\delta T(X(\sigma) + \delta A_\mu (X(\sigma))\partial X^\mu)(\sigma) + \cdots) \tag{3.2}$$

One could formally integrate this equation:

$$\Psi(X_*(\sigma)) \sim \int DX(\sigma) e^{S_{bound}+S_{bulk}} + \cdots \tag{3.3}$$

to get the wave function up to integration constants. The main motivation for (3.2) comes from the connection with the sigma model effective action. Note that convolutions of these three wave functions with the Witten's **CS** open string product [4] gives the expression for the partition function on the disk with free boundary conditions and the boundary action (3.1). This object is very close to the effective action for open string modes. Consider what gives this parameterization for the connection of the constant tachyon modes T_0 in sigma model with constant tachyon mode T^{CS} in **CS** string field theory. We have:

$$\frac{\partial T_0^{CS}}{\partial T_0} \sim e^{-\frac{1}{2}T_0} \tag{3.4}$$

(here $\frac{1}{2}$ comes from the integration of the tachyon over one-half of the boundary of the disk) Integrating this equation with the condition $T_0^{CS}(T_0 = 0) = 0$ we get

$$T_0^{CS} \sim e^{-\frac{1}{2}T_0} - 1 \tag{3.5}$$

In turn this gives the potential of the form:

$$V(T_0^{CS}) \sim (1 - 2\log(T_0^{CS} + 1))(T_0^{CS} + 1)^2 \tag{3.6}$$

Here the expansion around the "false" vacuum with $T_0^{CS} = 0$ could be compared with level approximation string field theory results [5],[3]. We plan to discuss the connection between sigma model and **CS** open string theory variables more thoroughly in [14].

4. Final remarks

We would like to conclude with few remarks not directly related to the subject of these notes, but we believe they bring right flavor to the discussion.

The interesting example of the importance of right choice of the fundamental fields is the theory of non-critical strings, mainly $c = 1$ model. The spectrum of the theory consists of the one massless scalar field and a set of discrete states [16].

The effect of these additional states on the S-matrix of the massless particles reduces to the non-invertible linear field redefinition:

$$\phi(p) \rightarrow \frac{\Gamma(1-2p)}{\Gamma(2p)}\phi(p) \tag{4.1}$$

All stringy phenomena (e.g. background independence) are hidden in this field redefinition (for details and extensive list of references see e.g. [17]

Finally note that according to the proposed scenario of the tachyon condensation the critical value of the tachyon field becomes infinite. We are going to discuss the implication of the topological classification of the tachyon vacua (see [18]) elsewhere [14].

Note added: After this work was finished the paper [19] appeared were the potential in (2.28) has been studied as a model example which mimics many expected properties of tachyon condensation. We claimed that this potential is in fact related to exact one via change of variables.

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